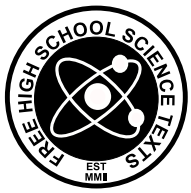

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The Free High School Science Texts: Textbooks for High School Students Studying the Sciences Mathematics Grades 10 - 12

Version 0
September 17, 2008

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Contents

Chapter 23

Solving Quadratic Inequalities - Grade 11

23.1 Introduction

Now that you know how to solve quadratic equations, you are ready to learn how to solve quadratic inequalities.

23.2 Quadratic Inequalities

A *quadratic inequality* is an inequality of the form

$$ax^2 + bx + c > 0$$
$$ax^2 + bx + c \geq 0$$
$$ax^2 + bx + c < 0$$
$$ax^2 + bx + c \leq 0$$

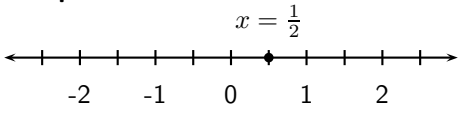
Solving a quadratic inequality corresponds to working out in what region the graph of a quadratic function lies above or below the x-axis.

Worked Example 112: Quadratic Inequality

Question: Solve the inequality $4x^2-4x+1 \leq 0$ and interpret the solution graphically.
Answer
Step 1 : Factorise the quadratic
Let $f(x) = 4x^2-4x+1$. Factorising this quadratic function gives $f(x) = (2x-1)^2$.
Step 2 : Re-write the original equation with factors

$$(2x-1)^2 \leq 0$$

Step 3 : Solve the equation
which shows that $f(x) = 0$ only when $x = \frac{1}{2}$.
Step 4 : Write the final answer
This means that the graph of $f(x) = 4x^2-4x+1$ touches the x-axis at $x = \frac{1}{2}$, but there are no regions where the graph is below the x-axis.
Step 5 : Graphical interpretation of solution

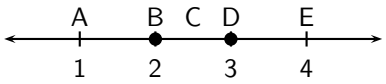


Worked Example 113: Solving Quadratic Inequalities

Question: Find all the solutions to the inequality $x^2 - 5x + 6 \geq 0$.
Answer
Step 1 : Factorise the quadratic
The factors of $x^2 - 5x + 6$ are $(x - 3)(x - 2)$.
Step 2 : Write the inequality with the factors

$$\begin{aligned} x^2 - 5x + 6 &\geq 0 \\ (x - 3)(x - 2) &\geq 0 \end{aligned}$$

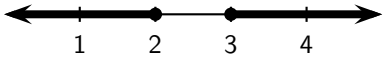
Step 3 : Determine which ranges correspond to the inequality
We need to figure out which values of x satisfy the inequality. From the answers we have five regions to consider.



Step 4 : Determine whether the function is negative or positive in each of the regions
Let $f(x) = x^2 - 5x + 6$. For each region, choose any point in the region and evaluate the function.

		$f(x)$	sign of $f(x)$
Region A	$x < 2$	$f(1) = 2$	$+$
Region B	$x = 2$	$f(2) = 0$	$+$
Region C	$2 < x < 3$	$f(2{,}5) = -2{,}5$	-
Region D	$x = 3$	$f(3) = 0$	$+$
Region E	$x > 3$	$f(4) = 2$	$+$

We see that the function is positive for $x \leq 2$ and $x \geq 3$.
Step 5 : Write the final answer and represent on a number line
We see that $x^2 - 5x + 6 \geq 0$ is true for $x \leq 2$ and $x \geq 3$.



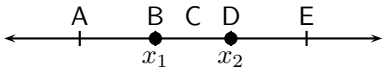
Worked Example 114: Solving Quadratic Inequalities

Question: Solve the quadratic inequality $-x^2 - 3x + 5 > 0$.
Answer
Step 1 : Determine how to approach the problem
Let $f(x) = -x^2 - 3x + 5$. $f(x)$ cannot be factorised so, use the quadratic formula to determine the roots of $f(x)$. The x-intercepts are solutions to the quadratic

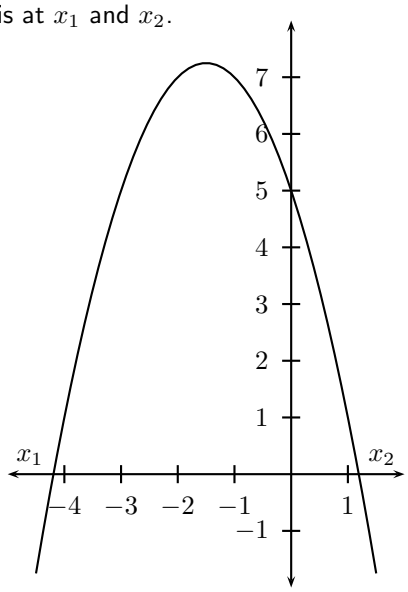
equation

$$\begin{aligned} -x^2 - 3x + 5 &= 0 \\ x^2 + 3x - 5 &= 0 \\ \therefore x &= \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-5)}}{2(1)} \\ &= \frac{-3 \pm \sqrt{29}}{2} \\ x_1 &= \frac{-3 - \sqrt{29}}{2} \\ x_2 &= \frac{-3 + \sqrt{29}}{2} \end{aligned}$$

Step 2 : Determine which ranges correspond to the inequality
We need to figure out which values of x satisfy the inequality. From the answers we have five regions to consider.



Step 3 : Determine whether the function is negative or positive in each of the regions
We can use another method to determine the sign of the function over different regions, by drawing a rough sketch of the graph of the function. We know that the roots of the function correspond to the x-intercepts of the graph. Let $g(x) = -x^2 - 3x + 5$. We can see that this is a parabola with a maximum turning point that intersects the x-axis at x_1 and x_2.



It is clear that $g(x) > 0$ for x_1
Step 4 : Write the final answer and represent the solution graphically
$-x^2 - 3x + 5 > 0$ for x_1

When working with an inequality where the variable is in the denominator, a different approach is needed.

Worked Example 115: Non-linear inequality with the variable in the denominator

Question: Solve $\frac{2}{x+3} \leq \frac{1}{x-3}$

Answer

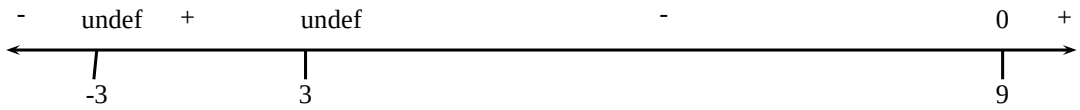
Step 1 : Subtract $\frac{1}{x-3}$ from both sides

$$\frac{2}{x+3} - \frac{1}{x-3} \leq 0$$

Step 2 : Simplify the fraction by finding LCD

$$\frac{2(x-3)-(x+3)}{(x+3)(x-3)} \leq 0$$
$$\frac{x-9}{(x+3)(x-3)} \leq 0$$

Step 3 : Draw a number line for the inequality



We see that the expression is negative for $x < -3$ or $3 < x \leq 9$.

Step 4 : Write the final answer

$$x < -3 \quad or \quad 3 < x \leq 9$$

23.3 End of Chapter Exercises

Solve the following inequalities and show your answer on a number line.

1. Solve: $x^2 - x < 12$.
2. Solve: $3x^2 > -x + 4$
3. Solve: $y^2 < -y - 2$
4. Solve: $-t^2 + 2t > -3$
5. Solve: $s^2 - 4s > -6$
6. Solve: $0 \geq 7x^2 - x + 8$
7. Solve: $0 \geq -4x^2 - x$
8. Solve: $0 \geq 6x^2$
9. Solve: $2x^22 + x + 6 \leq 0$
10. Solve for x if: $\frac{x}{x-3} < 2$ and $x \neq 3$.
11. Solve for x if: $\frac{4}{x-3} \leq 1$.

12. Solve for x if: $\frac{4}{(x-3)^2} < 1$.
13. Solve for x: $\frac{2x-2}{x-3} > 3$
14. Solve for x: $\frac{-3}{(x-3)(x+1)} < 0$
15. Solve: $(2x-3)^2 < 4$
16. Solve: $2x \leq \frac{15-x}{x}$
17. Solve for x: $\frac{x^2+3}{3x-2} \leq 0$
18. Solve: $x - 2 \geq \frac{3}{x}$
19. Solve for x: $\frac{x^2+3x-4}{5+x^4} \leq 0$
20. Determine all real solutions: $\frac{x-2}{3-x} \geq 1$

Appendix A

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