FHSST Authors

The Free High School Science Texts: Textbooks for High School Students Studying the Sciences Physics Grades 10 - 12

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Contents

Chapter 5

Transverse Pulses - Grade 10

5.1 Introduction

This chapter forms the basis of the discussion into mechanical waves. Waves are all around us, even though most of us are not aware of it. The most common waves are waves in the sea, but waves can be created in any container of water, ranging from an ocean to a tea-cup. Simply, a wave is moving energy.

5.2 What is a medium?

In this chapter, as well as in the following chapters, we will speak about waves moving in a medium. A medium is just the substance or material through which waves move. In other words the medium carries the wave from one place to another. The medium does not create the wave and the medium is not the wave. Air is a medium for sound waves, water is a medium for water waves and rock is a medium for earthquakes (which are also a type of wave). Air, water and rock are therefore examples of media (media is the plural of medium).

Definition: Medium
A medium is the substance or material in which a wave will move.

In each medium, the atoms that make up the medium are moved *temporarily* from their rest position. In order for a wave to travel, the different parts of the medium must be able to interact with each other.

5.3 What is a pulse?

Activity :: Investigation : Observation of Pulses
Take a heavy rope. Have two people hold the rope stretched out horizontally. Flick the rope at one end only once.

flick rope upwards at one end, once only

What happens to the disturbance that you created in the rope? Does it stay at the place where it was created or does it move down the length of the rope?

In the activity, we created a *pulse*. A pulse is a *single* disturbance that moves through a medium. A transverse pulse moves perpendicular to the medium. Figure 5.1 shows an example of a transverse pulse. In the activity, the rope or spring was held horizontally and the pulse moved the rope up and down. This was an example of a transverse pulse.

Definition: Pulse
A pulse is a single disturbance that moves through a medium.

5.3.1 Pulse Length and Amplitude

The amplitude of a pulse is a measurement of how far the medium is displaced from a position of rest. The pulse length is a measurement of how long the pulse is. Both these quantities are shown in Figure 5.1.

Definition: Amplitude
The amplitude of a pulse is a measurement of how far the medium is displaced from rest.

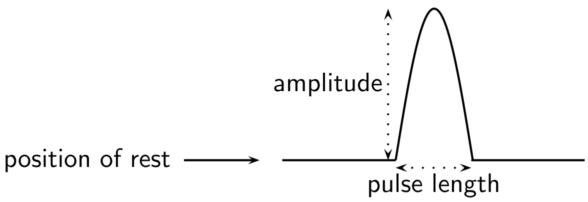
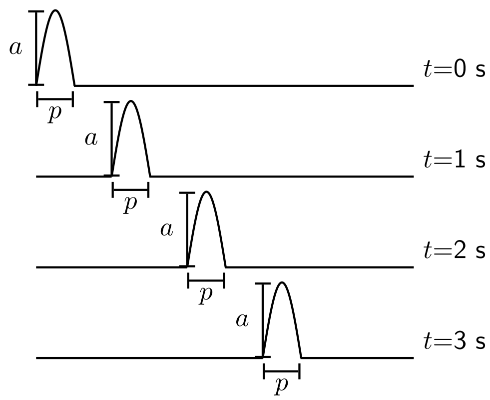


Figure 5.1: Example of a transverse pulse

Activity :: Investigation : Pulse Length and Amplitude
The graphs below show the positions of a pulse at different times.



Use your ruler to measure the lengths of a and p. Fill your answers in the table.

Time	a	p
$t = 0$ s		
$t = 1$ s		
$t = 2$ s		
$t = 3$ s		

What do you notice about the values of a and p?

In the activity, we found that the values for how high the pulse (a) is and how wide the pulse (p) is the same at different times. *Pulse length* and *amplitude* are two important quantities of a pulse.

5.3.2 Pulse Speed

Definition: Pulse Speed
Pulse speed is the distance a pulse travels in a specific time.

In Chapter 3 we saw that speed was defined as the distance travelled in a specified time. We can use the same definition of speed to calculate how fast a pulse travels. If the pulse travels a distance d in a time t, then the pulse speed v is:

$$v = \frac{d}{t}$$

Worked Example 22: Pulse Speed

Question: A pulse covers a distance of 2 m in 4 s on a heavy rope. Calculate the pulse speed.
Answer
Step 5 : Determine what is given and what is required
We are given:

- the distance travelled by the pulse: $d = 2$ m
- the time taken to travel 2 m: $t = 4$ s

We are required to calculate the speed of the pulse.
Step 6 : Determine how to approach the problem
We can use:

$$v = \frac{d}{t}$$

to calculate the speed of the pulse.
Step 7 : Calculate the pulse speed

$$\begin{aligned} v &= \frac{d}{t} \\ &= \frac{2\,\mathrm{m}}{4\,\mathrm{s}} \\ &= 0{,}5\,\mathrm{m}\cdot\mathrm{s}^{-1} \end{aligned}$$

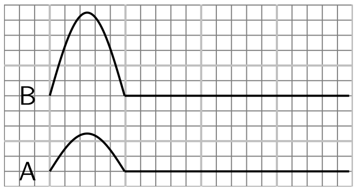
Step 8 : Write the final answer
The pulse speed is 0,5 m·s^{-1}.

Important: The pulse speed depends on the properties of the medium and not on the amplitude or pulse length of the pulse.

?

Exercise: Pulse Speed

1. A pulse covers a distance of 5 m in 15 seconds. Calculate the speed of the pulse.
2. A pulse has a speed of 5 cm.s^{-1}. How far does it travel in 2,5 seconds?
3. A pulse has a speed of 0,5 m·s^{-1}. How long does it take to cover a distance of 25 cm?
4. How long will it take a pulse moving at 0,25 m·s^{-1} to travel a distance of 20 m?
5. Examine the two pulses below and state which has the higher speed. Explain your answer.



6. Ocean waves do not bring more water onto the shore until the beach is completely submerged. Explain why this is so.

5.4 Graphs of Position and Velocity

When a pulse moves through a medium, there are two different motions: the motion of the particles of the medium and the motion of the pulse. These two motions are at right angles to each other when the pulse is transverse. Each motion will be discussed.

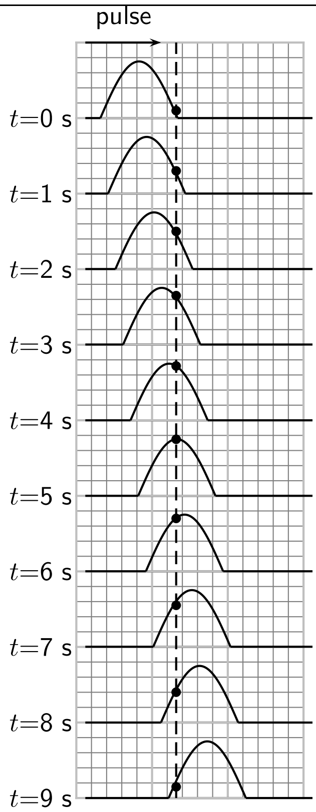
Consider the situation shown in Figure **??**. The dot represents one particle of the medium. We see that as the pulse moves to the right the particle only moves up and down.

5.4.1 Motion of a Particle of the Medium

First we consider the motion of a particle of the medium when a pulse moves through the medium. For the explanation we will zoom into the medium so that we are looking at the atoms of the medium. These atoms are connected to each other as shown in Figure 5.2.

Figure 5.2: Particles in a medium.

When a pulse moves through the medium, the particles in the medium **only** move up and down. We can see this in the figure below which shows the motion of a single particle as a pulse moves through the medium.



Important: A particle in the medium **only** moves up and down when a transverse pulse moves through the medium. The pulse moves from left to right (or right to left). The motion of the particle is perpendicular to the motion of a transverse pulse.

If you consider the motion of the particle as a function of time, you can draw a graph of position vs. time and velocity vs. time.

Activity :: Investigation : Drawing a position-time graph

1. Study Figure **??** and complete the following table:

time (s)	0	1	2	3	4	5	6	7	8	9
position (cm)										

2. Use your table to draw a graph of position vs. time for a particle in a medium.

The position vs. time graph for a particle in a medium when a pulse passes through the medium is shown in Figure 5.3

Activity :: Investigation : Drawing a velocity-time graph

1. Study Figure 5.3 and Figure 5.4 and complete the following table:

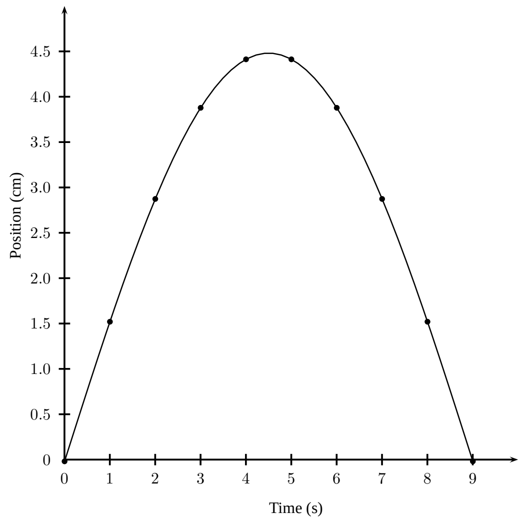


Figure 5.3: Position against Time graph of a particle in the medium through which a transverse pulse is travelling.

time (s)	0	1	2	3	4	5	6	7	8	9
velocity (cm.s^{-1})										

2. Use your table to draw a graph of velocity vs time for a particle in a medium.

The velocity vs. time graph far a particle in a medium when a pulse passes through the medium is shown in Figure 5.4.

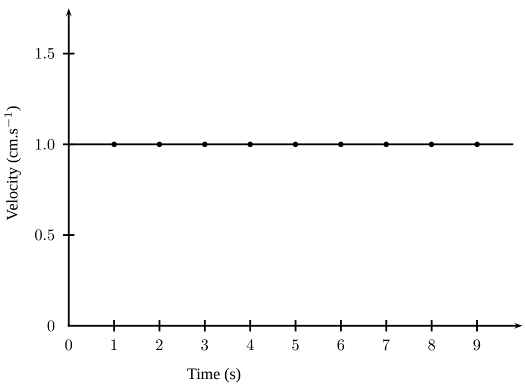


Figure 5.4: Velocity against Time graph of a particle in the medium through which a transverse pulse is travelling.

5.4.2 Motion of the Pulse

The motion of the pulse is much simpler than the motion of a particle in the medium.

Important: A point on a transverse pulse, eg. the peak, **only** moves in the direction of the motion of the pulse.

Worked Example 23: Transverse pulse through a medium

Question:

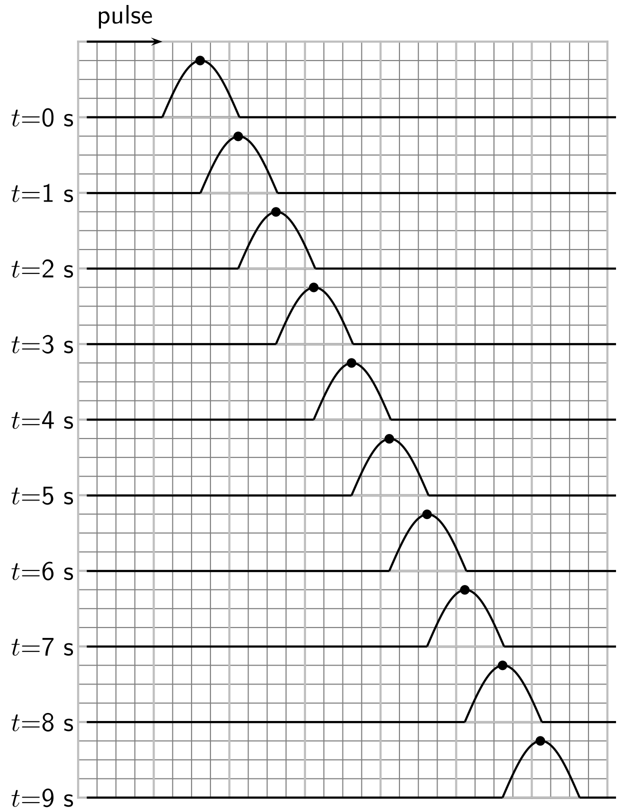


Figure 5.5: Position of the peak of a pulse at different times (since we know the shape of the pulse does not change we can look at only one point on the pulse to keep track of its position, the peak for example). The pulse moves to the right as shown by the arrow.

Given the series of snapshots of a transverse pulse moving through a medium, depicted in Figure 5.5, do the following:

- draw up a table of time, position and velocity,
- plot a position vs. time graph,
- plot a velocity vs. time graph.

Answer

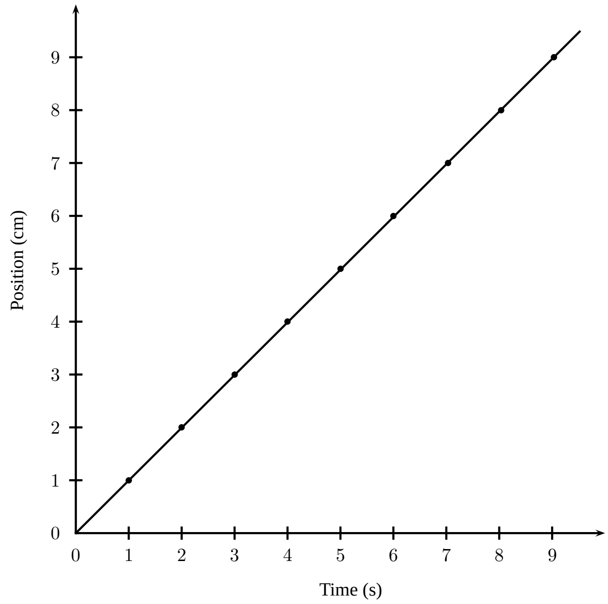
Step 1 : Interpreting the figure

Figure 5.5 shows the motion of a pulse through a medium and a dot to indicate the same position on the pulse. If we follow the dot, we can draw a graph of position vs time for a pulse. At t = 0 s the dot is at 0cm. At t = 1 s the dot is 1 cm away from its original postion. At t = 2 s the dot is 2 cm away from its original postion, and so on.

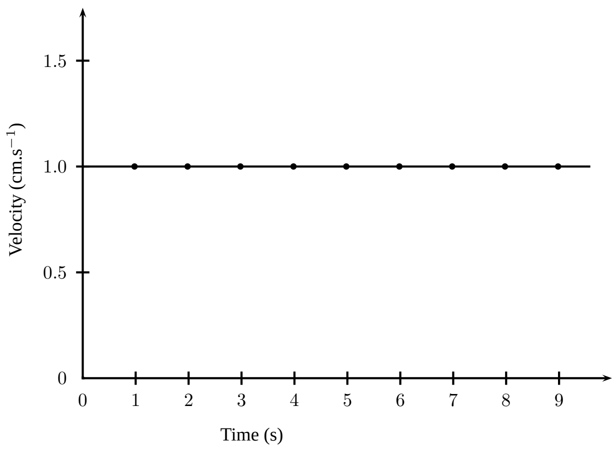
Step 2 : We can draw the following table:

time (s)	0	1	2	3	4	5	6	7	8	9
position (cm)										
velocity (cm.s^{-1})										

Step 3 : A graph of position vs time is drawn as is shown in the figure.



Step 4 : Similarly, a graph of velocity vs time is drawn and is shown in the figure below.



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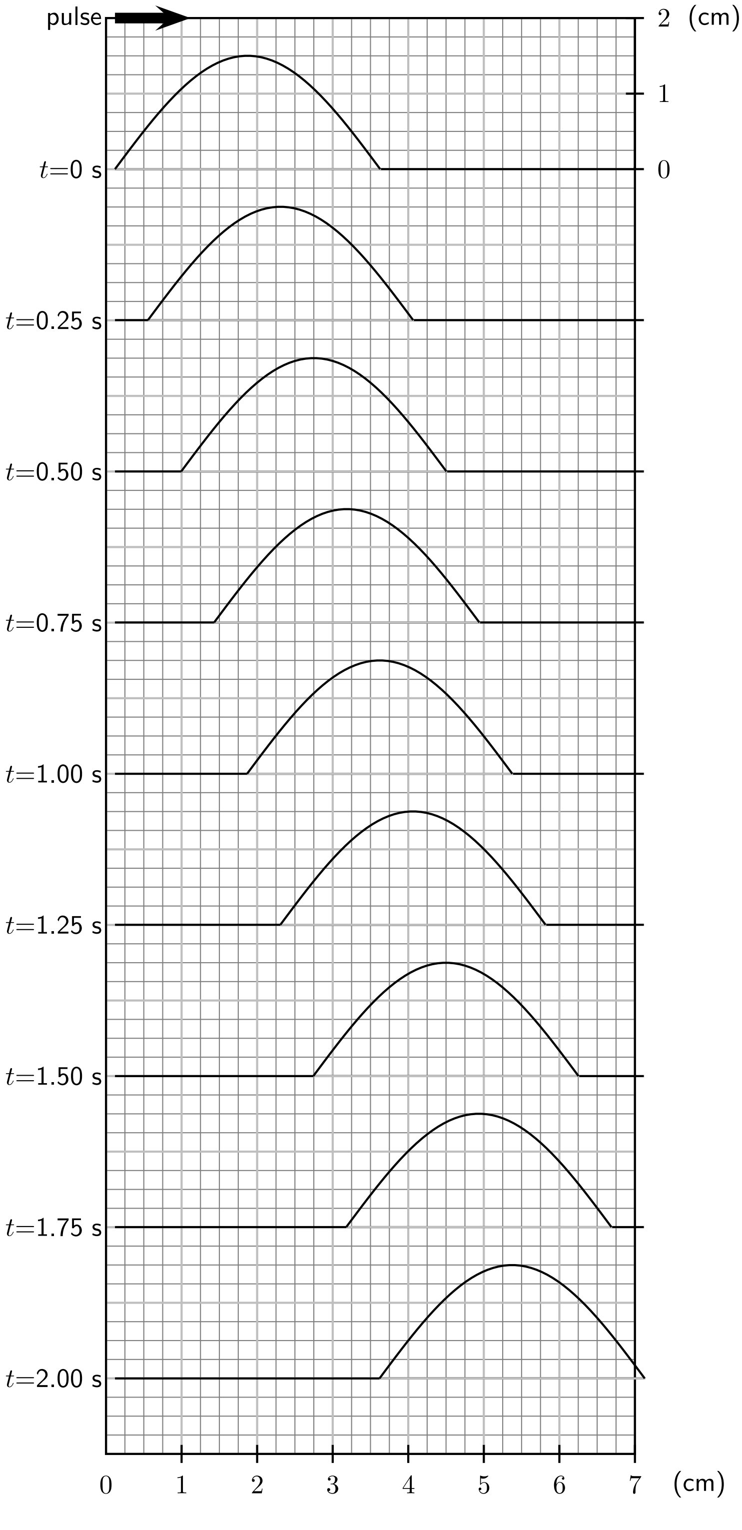
Exercise: Travelling Pulse

1. A pulse is passed through a rope and the following pictures were obtained for each time interval:



(a) Complete the following table for a particle in the medium:

time (s)	0,00	0,25	0,50	0,75	1,00	1,25	1,50	1,75	2,00
position (mm)									
velocity (mm.s$^-1$)									

(b) Draw a position vs. time graph for the motion of a particle in the medium.

(c) Draw a velocity vs. time graph for the motion of a particle in the medium.

(d) Draw a position vs. time graph for the motion of the pulse through the rope.

(e) Draw a velocity vs. time graph for the motion of the pulse through the rope.

5.5 Transmission and Reflection of a Pulse at a Boundary

What happens when a pulse travelling in one medium finds that medium is joined to another?

Activity :: Investigation : Two ropes
Find two different ropes and tie both ropes together. Hold the joined ropes horizontally and create a pulse by flicking the rope up and down. What happens to the pulse when it encounters the join?

When a pulse meets a boundary between two media, part of the pulse is reflected and part of it is transmitted. You will see that in the thin rope the pulse moves back (is reflected). The pulse is also passed on (transmitted) to the thick rope and it moves away from the boundary.

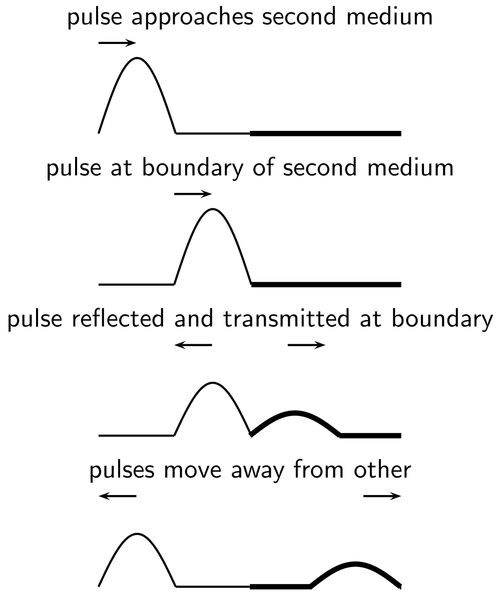


Figure 5.6: Reflection and transmission of a pulse at the boundary between two media.

When a pulse is transmitted from one medium to another, like from a thin rope to a thicker one, the pulse will change where it meets the boundary of the two mediums (for example where the ropes are joined). When a pulse moves from a thin rope to a thicker one, the speed of the pulse will decrease. The pulse will move slower and the pulse length will increase.

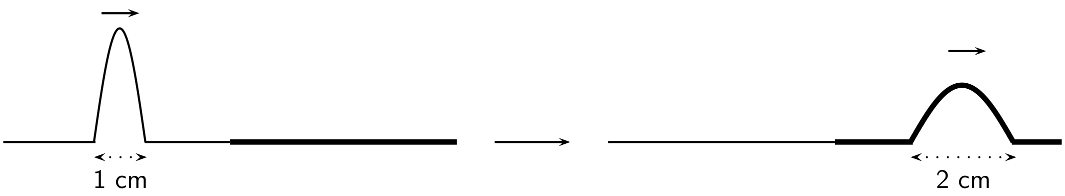


Figure 5.7: Reflection and transmission of a pulse at the boundary between two media.

Figure 5.8: Reflection and transmission of a pulse at the boundary between two media.

When a pulse moves from a thick rope to a thinner one, the opposite happens. The pulse speed will increase and the pulse length will decrease.

When the speed of the pulse increases, the pulse length will decrease. If the speed decreases, the pulse length will increase. The **incident pulse** is the one that arrives at the boundary. The **reflected pulse** is the one that moves back, away from the boundary. The **transmitted pulse** is the one that moves into the new medium, away from the boundary.

Exercise: Pulses at a Boundary I

1. Fill in the blanks or select the correct answer: A pulse in a heavy rope is traveling towards the boundary with a thin piece of string.
   (a) The reflected pulse in the heavy rope **will/will not** be inverted because ____________.
   (b) The speed of the transmitted pulse will be **greater than/less than/the same as** the speed of the incident pulse.
   (c) The speed of the reflected pulse will be **greater than/less than/the same as** the speed of the incident pulse.
   (d) The pulse length of the transmitted pulse will be **greater than/less than/the same as** the pulse length of the incident pulse.
   (e) The frequency of the transmitted pulse will be **greater than/less than/the same as** the frequency of the incident pulse.
2. A pulse in a light string is traveling towards the boundary with a heavy rope.
   (a) The reflected pulse in the light rope **will/will not** be inverted because _________.
   (b) The speed of the transmitted pulse will be **greater than/less than/the same as** the speed of the incident pulse.
   (c) The speed of the reflected pulse will be **greater than/less than/the same as** the speed of the incident pulse.
   (d) The pulse length of the transmitted pulse will be **greater than/less than/the same as** the pulse length of the incident pulse.

5.6 Reflection of a Pulse from Fixed and Free Ends

Let us now consider what happens to a pulse when it reaches the end of a medium. The medium can be fixed, like a rope tied to a wall, or it can be free, like a rope tied loosely to a pole.

5.6.1 Reflection of a Pulse from a Fixed End

Activity :: Investigation : Reflection of a Pulse from a Fixed End
Tie a rope to a wall or some other object that cannot move. Create a pulse in the rope by flicking one end up and down. Observe what happens to the pulse when it reaches the wall.

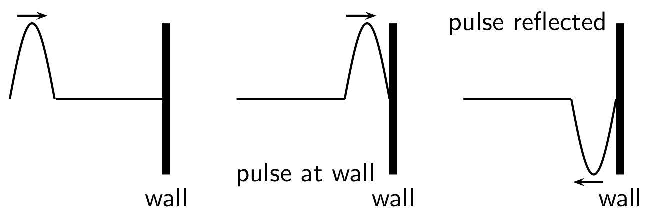


Figure 5.9: Reflection of a pulse from a fixed end.

When the end of the medium is fixed, for example a rope tied to a wall, a pulse reflects from the fixed end, but the pulse is inverted (i.e. it is upside-down). This is shown in Figure 5.9.

5.6.2 Reflection of a Pulse from a Free End

Activity :: Investigation : Reflection of a Pulse from a Free End
Tie a rope to a pole in such a way that the rope can move up and down the pole. Create a pulse in the rope by flicking one end up and down. Observe what happens to the pulse when it reaches the pole.

When the end of the medium is free, for example a rope tied loosely to a pole, a pulse reflects from the free end, but the pulse is not inverted. This is shown in Figure 5.10. We draw the free end as a ring around the pole. The ring will move up and down the pole, while the pulse is reflected away from the pole.

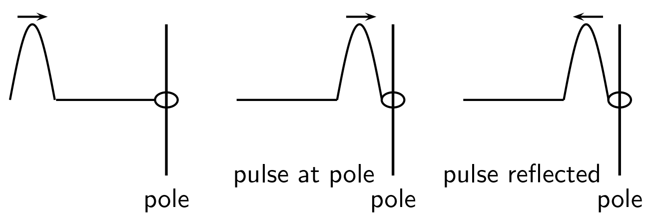


Figure 5.10: Reflection of a pulse from a free end.

Important: The fixed and free ends that were discussed in this section are examples of *boundary conditions*. You will see more of boundary conditions as you progress in the Physics syllabus.

Exercise: Pulses at a Boundary II

1. A rope is tied to a tree and a single pulse is generated. What happens to the pulse as it reaches the tree? Draw a diagram to explain what happens.

2. A rope is tied to a ring that is loosely fitted around a pole. A single pulse is sent along the rope. What will happen to the pulse as it reaches the pole? Draw a diagram to explain your answer.

5.7 Superposition of Pulses

Two or more pulses can pass through the same medium at that same time. The resulting pulse is obtained by using the *principle of superposition*. The principle of superposition states that the effect of the pulses is the sum of their individual effects. After pulses pass through each other, each pulse continues along its original direction of travel, and their original amplitudes remain unchanged.

Constructive interference takes place when two pulses meet each other to create a larger pulse. The amplitude of the resulting pulse is the sum of the amplitudes of the two initial pulses. This is shown in Figure 5.11.

Definition: Constructive interference is when two pulses meet, resulting in a bigger pulse.

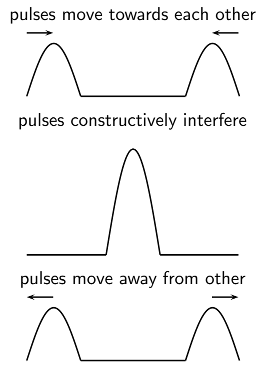


Figure 5.11: Superposition of two pulses: constructive interference.

Destructive interference takes place when two pulses meet and cancel each other. The amplitude of the resulting pulse is the sum of the amplitudes of the two initial pulses, but the one amplitude will be a negative number. This is shown in Figure 5.12. In general, amplitudes of individual pulses add together to give the amplitude of the resultant pulse.

Definition: Destructive interference is when two pulses meet, resulting in a smaller pulse.

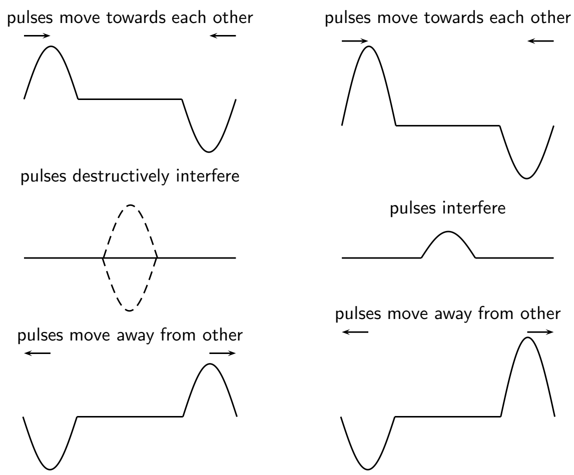
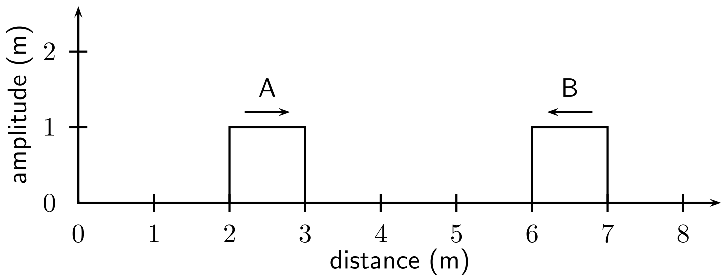


Figure 5.12: Superposition of two pulses. The left-hand series of images demonstrates destructive interference, since the pulses cancel each other. The right-hand series of images demonstrate a partial cancelation of two pulses, as their amplitudes are not the same in magnitude.

Worked Example 24: Superposition of Pulses

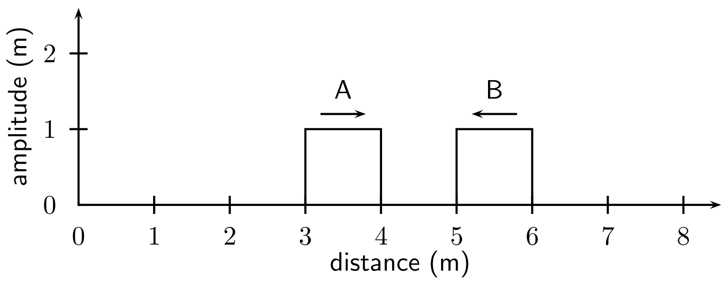
Question: The two pulses shown below approach each other at 1 m·s^{-1}. Draw what the waveform would look like after 1 s, 2 s and 5 s.



Answer

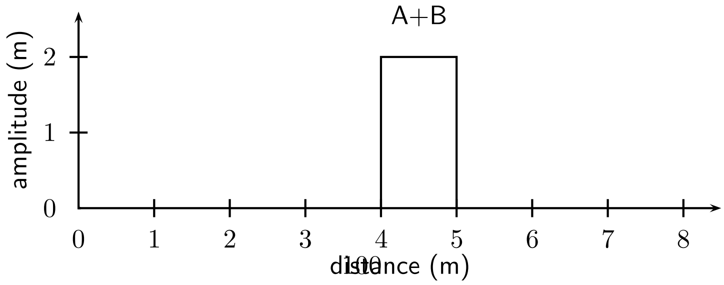
Step 1 : After 1 s

After 1 s, pulse A has moved 1 m to the right and pulse B has moved 1 m to the left.



Step 2 : After 2 s

After 1 s more, pulse A has moved 1 m to the right and pulse B has moved 1 m to the left.

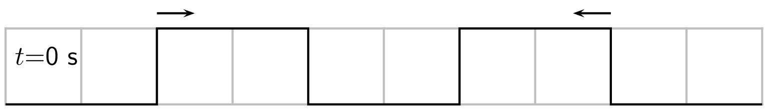


Step 3 : After 5 s

Important: The idea of superposition is one that occurs often in physics. You will see *much, much more* of superposition!

Exercise: Superposition of Pulses

1. For each of the following pulses, draw the resulting wave forms after 1 s, 2 s, 3 s, 4 s and 5 s. Each pulse is travelling at 1 m·s^{-1}. Each block represents 1 m.

(a)


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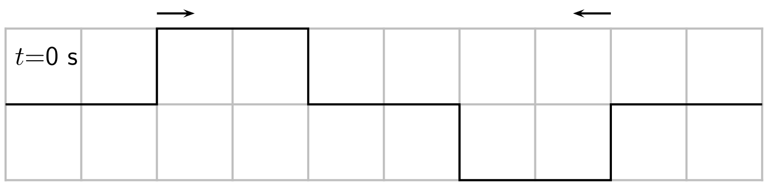
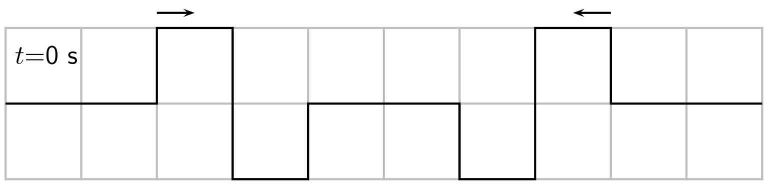
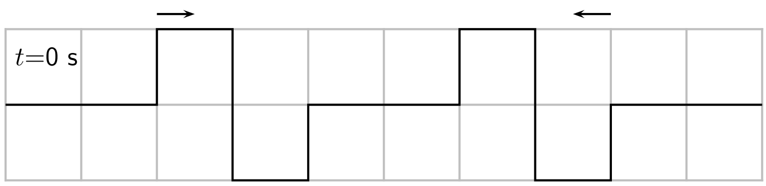


(c)


(d)


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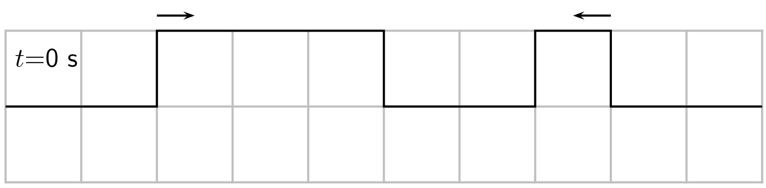

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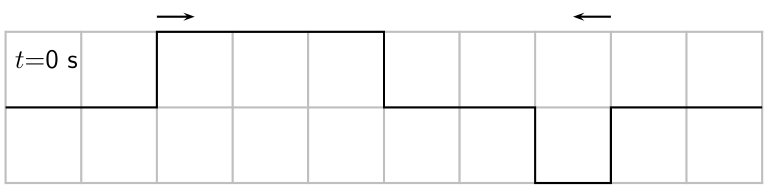
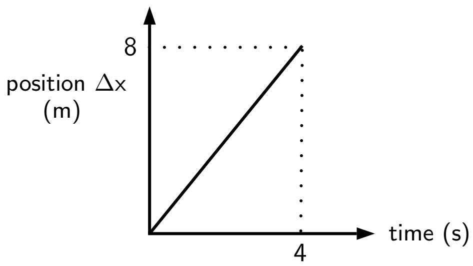


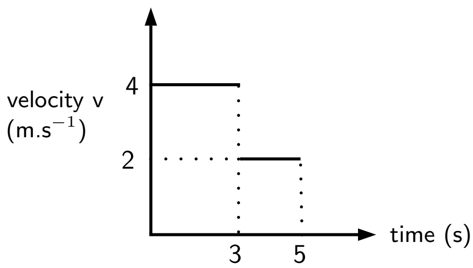
2. (a) What is superposition of waves?
   (b) What is constructive interference? Use the letter "c" to indicate where constructive interference took place in each of your answers for question 1. Only look at diagrams for $t = 3$ s.
   (c) What is destructive interference? Use the letter "d" to indicate where destructive interference took place in each of your answers for question 1. Only look at diagrams for $t = 2$ s.

5.8 Exercises - Transverse Pulses

1. A heavy rope is flicked upwards, creating a single pulse in the rope. Make a drawing of the rope and indicate the following in your drawing:
   (a) The direction of motion of the pulse
   (b) Amplitude
   (c) Pulse length
   (d) Position of rest
2. A pulse has a speed of 2,5m.s^{-1}. How far will it have travelled in 6s?
3. A pulse covers a distance of 75cm in 2,5s. What is the speed of the pulse?
4. How long does it take a pulse to cover a distance of 200mm if its speed is 4m.s^{-1}?
5. The following position-time graph for a pulse in a slinky spring is given. Draw an accurate sketch graph of the velocity of the pulse against time.

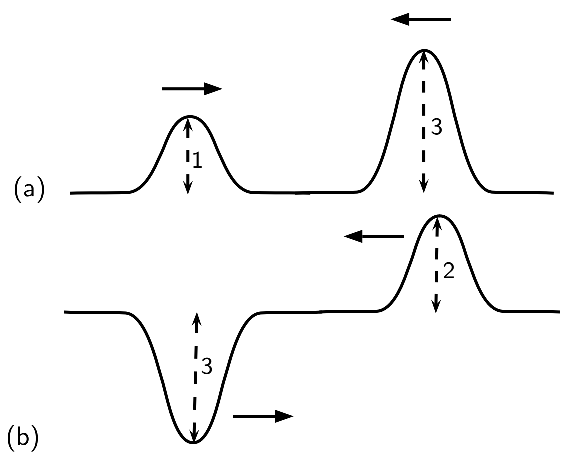


6. The following velocity-time graph for a particle in a medium is given. Draw an accurate sketch graph of the position of the particle vs. time.



7. Describe what happens to a pulse in a slinky spring when:
   (a) the slinky spring is tied to a wall.
   (b) the slinky spring is loose, i.e. not tied to a wall.

   (Draw diagrams to explain your answers.)
8. The following diagrams each show two approaching pulses. Redraw the diagrams to show what type of interference takes place, and label the type of interference.



9. Two pulses, A and B, of identical frequency and amplitude are simultaneously generated in two identical wires of equal mass and length. Wire A is, however, pulled tighter than wire. Which pulse will arrive at the other end first, or will they both arrive at the same time?

Appendix A

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