



FHSST Authors

**The Free High School Science Texts:
Textbooks for High School Students
Studying the Sciences
Physics
Grades 10 - 12**

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Chapter 28

Electrodynamics - Grade 12

28.1 Introduction

In Grade 11 you learnt how a magnetic field is generated around a current carrying conductor. You also learnt how a current is generated in a conductor that moves in a magnetic field. This chapter describes how conductors moved in a magnetic field are applied in the real-world.

28.2 Electrical machines - generators and motors

We have seen that when a conductor is moved in a magnetic field or when a magnet is moved near a conductor, such that the magnetic field is not parallel to the conductor, a current flows in the conductor. The amount of current depends on the speed at which the conductor experiences a changing magnetic field, the number of turns of the conductor and the orientation of the plane of the conductor with respect to the magnetic field. The effect of the orientation of the conductor with respect to the magnetic field is shown in Figure 28.1.

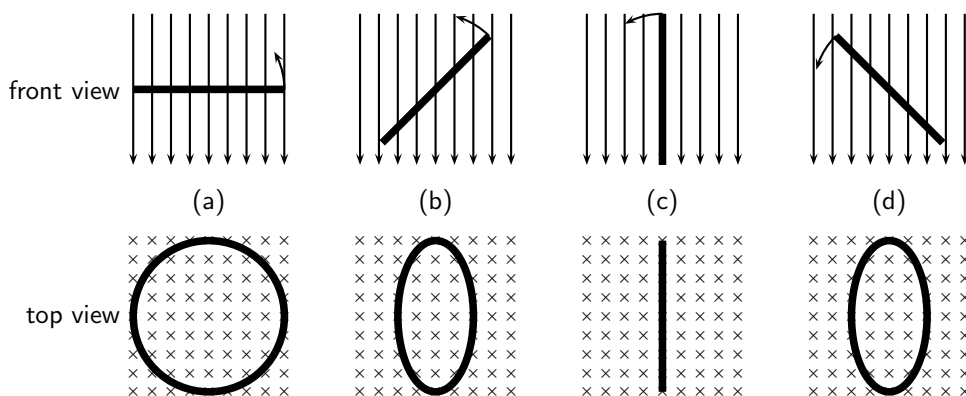


Figure 28.1: Series of figures showing that the magnetic flux through a conductor is dependent on the angle that the plane of the conductor makes with the magnetic field. The greatest flux passes through the conductor when the plane of the conductor is perpendicular to the magnetic field lines as in (a). The number of field lines passing through the conductor decreases, as the conductor rotates until it is parallel to the magnetic field.

If the current flowing in the conductor were plotted as a function of the angle between the plane of the conductor and the magnetic field, then the current would vary as shown in Figure 28.2. The current alternates about the zero value and is also known as an *alternating current* (abbreviated AC).

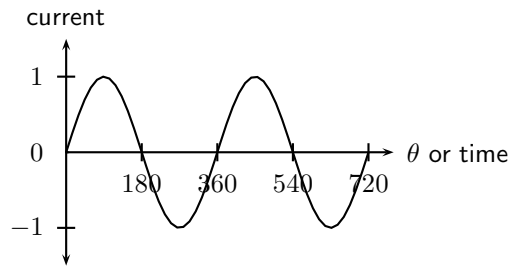


Figure 28.2: Variation of current as angle of plane of conductor with the magnetic field changes.

28.2.1 Electrical generators

AC generator

The principle of rotating a conductor in a magnetic field is used in electricity generators. A generator converts mechanical energy into electrical energy.



Definition: Generator

A generator converts mechanical energy into electrical energy.

The layout of an AC generator is shown in Figure 28.3. The conductor in the shape of a coil is connected to a ring. The conductor is then manually rotated in the magnetic field generating an alternating emf. The slip rings are connected to the load via brushes.

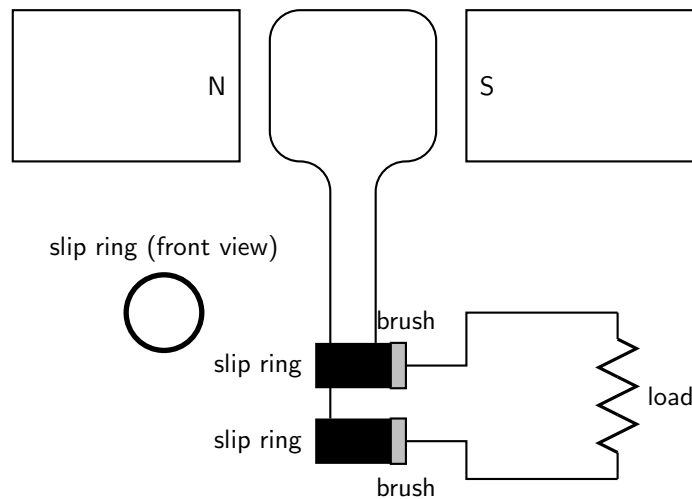
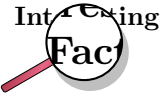


Figure 28.3: Layout of an alternating current generator.

If a machine is constructed to rotate a magnetic field around a set of stationary wire coils with the turning of a shaft, AC voltage will be produced across the wire coils as that shaft is rotated, in accordance with Faraday's Law of electromagnetic induction. This is the basic operating principle of an AC generator.

In an AC generator the two ends of the coil are each attached to a slip ring that makes contact with brushes as the coil turns. The direction of the current changes with every half turn of the coil. As one side of the loop moves to the other pole of the magnetic field, the current in it changes direction. The two slip rings of the AC generator allow the current to change directions and become alternating current.



AC generators are also known as alternators. They are found in motor cars to charge the car battery.

DC generator

A DC generator is constructed the same way as an AC generator except that there is one slip ring which is split into two pieces, called a commutator, so the current in the external circuit does not change direction. The layout of a DC generator is shown in Figure 28.4. The split-ring commutator accommodates for the change in direction of the current in the loop, thus creating DC current going through the brushes and out to the circuit.

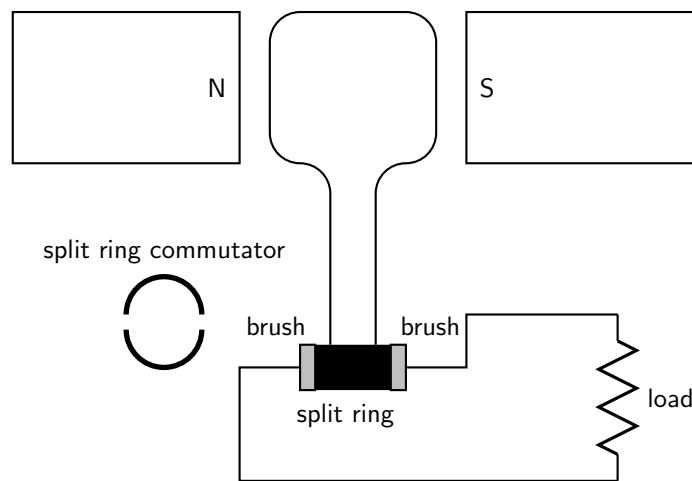


Figure 28.4: Layout of a direct current generator.

The shape of the emf from a DC generator is shown in Figure 28.5. The emf is not steady but is more or less the positive halves of a sine wave.

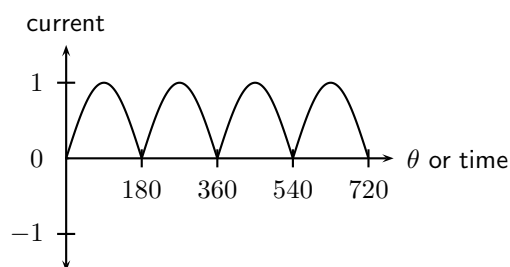


Figure 28.5: Variation of emf in a DC generator.

AC versus DC generators

The problems involved with making and breaking electrical contact with a moving coil should be obvious (sparking and heat), especially if the shaft of the generator is revolving at high speed. If the atmosphere surrounding the machine contains flammable or explosive vapors, the practical problems of spark-producing brush contacts are even greater.

An AC generator (alternator) does not require brushes and commutators to work, and so is immune to these problems experienced by DC generators. The benefits of AC over DC with regard to generator design is also reflected in electric motors. While DC motors require the use

of brushes to make electrical contact with moving coils of wire, AC motors do not. In fact, AC and DC motor designs are very similar to their generator counterparts. The AC motor being dependent upon the reversing magnetic field produced by alternating current through its stationary coils of wire to rotate the rotating magnet around on its shaft, and the DC motor being dependent on the brush contacts making and breaking connections to reverse current through the rotating coil every 1/2 rotation (180 degrees).

28.2.2 Electric motors

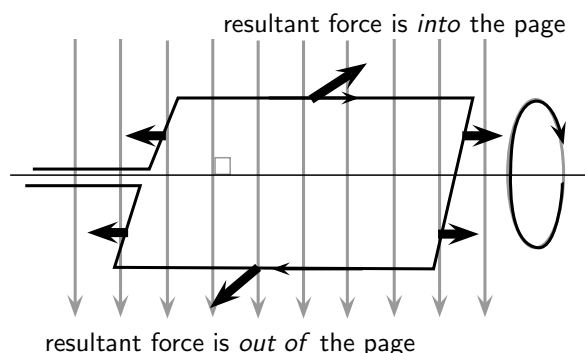
The basic principles of operation for a motor are the same as that of a generator, except that a motor converts electrical energy into mechanical energy.



Definition: Motor

An electric motor converts electrical energy into mechanical energy.

Both motors and generators can be explained in terms of a coil that rotates in a magnetic field. In a generator the coil is attached to an external circuit and it is mechanically turned, resulting in a changing flux that induces an emf. In a motor, a current-carrying coil in a magnetic field experiences a force on both sides of the coil, creating a torque which makes it turn. Any coil carrying current can feel a force in a magnetic field, the force is the Lorentz force on the moving charges in the conductor. We know that if the coil is parallel to the magnetic field then the Lorentz force will be zero. The charge of opposite sides of the coil will be in opposite directions because the charges are moving in opposite directions. This means the coil will rotate.



Instead of rotating the loops through a magnetic field to create electricity, a current is sent through the wires, creating electromagnets. The outer magnets will then repel the electromagnets and rotate the shaft as an electric motor. If the current is AC, the two slip rings are required to create an AC motor. An AC motor is shown in Figure 28.6

If the current is DC, split-ring commutators are required to create a DC motor. This is shown in Figure 28.7.

28.2.3 Real-life applications

Cars

A car contains an alternator that charges up its battery power the car's electric system when its engine is running. Alternators have the great advantage over direct-current generators of not using a commutator, which makes them simpler, lighter, less costly, and more rugged than a DC generator.

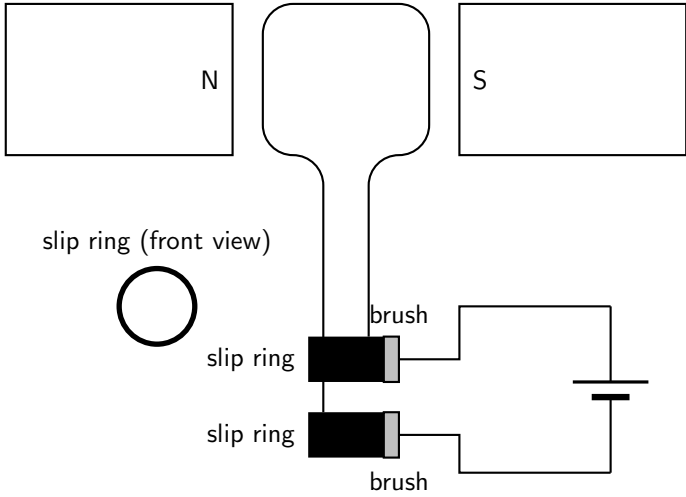


Figure 28.6: Layout of an alternating current motor.

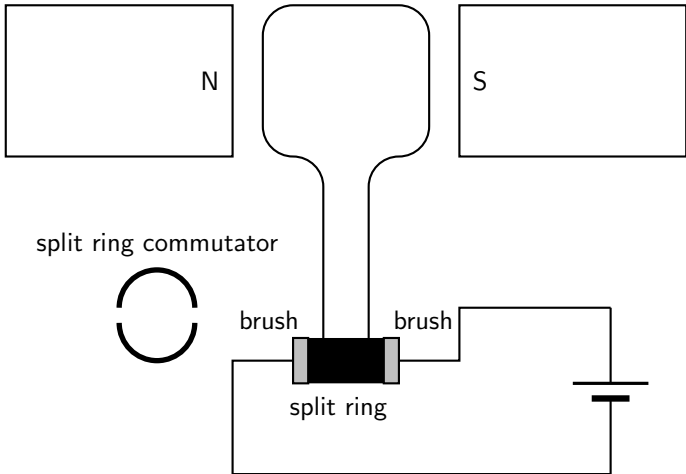


Figure 28.7: Layout of a direct current motor.

Activity :: Research Topic : Alternators

Try to find out the different ampere values produced by alternators for different types of machines. Compare these to understand what numbers make sense in the real world. You will find different numbers for cars, trucks, buses, boats etc. Try to find out what other machines might have alternators.

A car also contains a DC electric motor, the starter motor, to turn over the engine to start it. A starter consists of the very powerful DC electric motor and starter solenoid that is attached to the motor. A starter motor requires very high current to crank the engine, that's why it's connected to the battery with large cables.

Electricity Generation

AC generators are mainly used in the real-world to generate electricity.

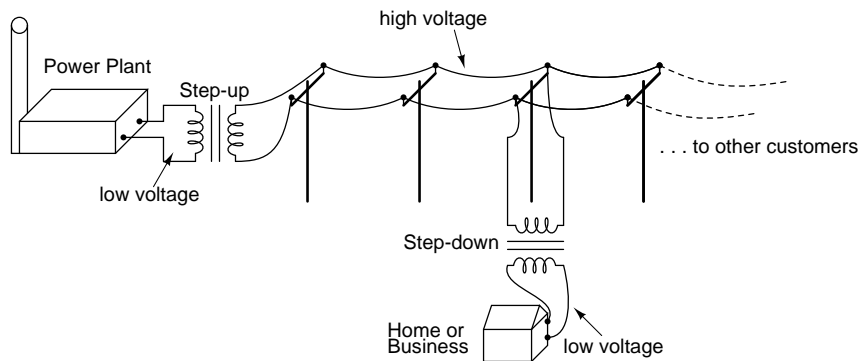


Figure 28.8: AC generators are used at the power plant to generate electricity.

28.2.4 Exercise - generators and motors

1. State the difference between a generator and a motor.
2. Use Faraday's Law to explain why a current is induced in a coil that is rotated in a magnetic field.
3. Explain the basic principle of an AC generator in which a coil is mechanically rotated in a magnetic field. Draw a diagram to support your answer.
4. Explain how a DC generator works. Draw a diagram to support your answer. Also, describe how a DC generator differs from an AC generator.
5. Explain why a current-carrying coil placed in a magnetic field (but not parallel to the field) will turn. Refer to the force exerted on moving charges by a magnetic field and the torque on the coil.
6. Explain the basic principle of an electric motor. Draw a diagram to support your answer.
7. Give examples of the use of AC and DC generators.
8. Give examples of the use of motors

28.3 Alternating Current

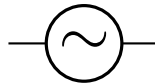
Most students of electricity begin their study with what is known as *direct current* (DC), which is electricity flowing in a constant direction. DC is the kind of electricity made by a battery, with definite positive and negative terminals).

However, we have seen that the electricity produced by a generator alternates and is therefore known as *alternating current* (AC). The main advantage to AC is that the voltage can be changed using transformers. That means that the voltage can be stepped up at power stations to a very high voltage so that electrical energy can be transmitted along power lines at low current and therefore experience low energy loss due to heating. The voltage can then be stepped down for use in buildings and street lights.



In South Africa alternating current is generated at a frequency of 50 Hz.

The circuit symbol for alternating current is:



Graphs of voltage against time and current against time for an AC circuit are shown in Figure 28.9

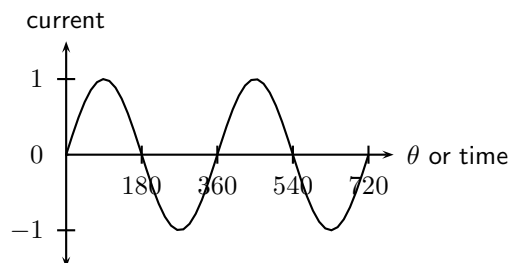


Figure 28.9: Graph of current or voltage in an AC circuit.

In a DC circuit the current and voltage are constant. In an AC circuit the current and voltage vary with time. The value of the current or voltage at any specific moment in time is called the *instantaneous current or voltage* and is calculated as follows:

$$i = I_{max} \sin(2\pi ft)$$

$$v = V_{max} \sin(2\pi ft)$$

i is the instantaneous current. I_{max} is the maximum current. v is the instantaneous voltage. V_{max} is the maximum voltage. f is the frequency of the AC and t is the time at which the instantaneous current or voltage is being calculated.

This average value we use for AC is known as the root mean square (rms) average. This is defined as:

$$I_{rms} = \frac{I_{max}}{\sqrt{2}}$$

$$V_{rms} = \frac{V_{max}}{\sqrt{2}}$$

Since AC varies sinusoidally, with as much positive as negative, doing a straight average would get you zero for the average voltage. The rms value by-passes this problem.

28.3.1 Exercise - alternating current

1. Explain the advantages of alternating current.
2. Write expressions for the current and voltage in an AC circuit.
3. Define the rms (root mean square) values for current and voltage for AC.
4. What is the period of the AC generated in South Africa?
5. If the mains supply is 200 V AC, calculate rms voltage.
6. Draw a graph of voltage vs time and current vs time for an AC circuit.

28.4 Capacitance and inductance

Capacitors and inductors are found in many circuits. Capacitors store an electric field, and are used as temporary power sources as well as minimize power fluctuations in major circuits. Inductors work in conjunction with capacitors for electrical signal processing. Here we explain the physics and applications of both.

28.4.1 Capacitance

You have learnt about capacitance and capacitors in Grade 11. Please read through section 17.5 to recap what you learnt about capacitance in a DC circuit.

In this section you will learn about capacitance in an AC circuit. A capacitor in an AC circuit has *reactance*. Reactance in an AC circuit plays a similar role to resistance in a DC circuit.

The reactance of a capacitor X_C is defined as:

$$X_C = \frac{1}{2\pi fC}$$

where C is the capacitance and f is the AC frequency.

If we examine the equation for the reactance of a capacitor, we see that the frequency is in the denominator. Therefore, when the frequency is low, the capacitive reactance is very high. This is why a capacitor blocks the flow of DC and low frequency AC because its reactance increases with decreasing frequency.

When the frequency is high, the capacitive reactance is low. This is why a capacitor allows the flow of high frequency AC because its reactance decreases with increasing frequency.

28.4.2 Inductance

An inductor is a passive electrical device used in electrical circuits for its property of inductance. An inductor is usually made as a coil (or solenoid) of conducting material, typically copper wire, wrapped around a core either of air or of ferromagnetic material.

Electrical current through the conductor creates a magnetic flux proportional to the current. A change in this current creates a change in magnetic flux that, in turn, generates an emf that acts to oppose this change in current.

Inductance (measured in henries, symbol H) is a measure of the generated emf for a unit change in current. For example, an inductor with an inductance of 1 H produces an emf of 1 V when the current through the inductor changes at the rate of $1 \text{ A}\cdot\text{s}^{-1}$.

The inductance of an inductor is determined by several factors:

- the shape of the coil; a short, fat coil has a higher inductance than one that is thin and tall.
- the material that conductor is wrapped around.
- how the conductor is wound; winding in opposite directions will cancel out the inductance effect, and you will have only a resistor.

The inductance of a solenoid is defined by:

$$L = \frac{\mu_0 AN^2}{l}$$

where μ_0 is the permeability of the core material (in this case air), A is the cross-sectional area of the solenoid, N is the number of turns and l is the length of the solenoid.



Definition: Permeability

Permeability is the property of a material which describes the magnetisation developed in that material when excited by a source.



The permeability of free space is $4\pi \times 10^{-7}$ henry per metre.



Worked Example 177: Inductance I

Question:

Answer

Determine the inductance of a coil with a core material of air. A cross-sectional area of $0,3\text{m}^2$, with 1000 turns and a length of $0,1\text{ m}$

Step 1 : Determine how to approach the problem

We are calculating inductance, so we use the equation:

$$L = \frac{\mu_0 AN^2}{l}$$

The permeability is that for free space: $4\pi \times 10^{-7}$ henry per metre.

Step 2 : Solve the problem

$$\begin{aligned} L &= \frac{\mu_0 AN^2}{l} \\ &= \frac{(4\pi \times 10^{-7})(0,3)(1000)}{0,1} \\ &= 3,8 \times 10^{-3} \text{ H/m} \end{aligned}$$

Step 3 : Write the final answer

The inductance of the coil is $3,8 \times 10^{-3} \text{ H/m}$.



Worked Example 178: Inductance II

Question: Calculate the inductance of a 5 cm long solenoid with a diameter of 4 mm and 2000 turns.

Answer

Again this is an inductance problem, so we use the same formula as the worked example above.

$$r = \frac{4 \text{ mm}}{2} = 2 \text{ mm} = 0,002 \text{ m}$$

$$A = \pi r^2 = \pi \times 0,002^2$$

$$\begin{aligned} L &= \frac{\mu_0 AN^2}{l} \\ &= \frac{4\pi \times 10^{-7} \times 0,002^2 \times \pi \times 2000^2}{0,05} \\ &= 0,00126 \text{ H} \\ &= 1,26 \text{ mH} \end{aligned}$$

An inductor in an AC circuit also has a reactance, X_L that is defined by:

$$X_L = 2\pi fL$$

where L is the inductance and f is the frequency of the AC.

If we examine the equation for the reactance of an inductor, we see that inductive reactance increases with increasing frequency. Therefore, when the frequency is low, the inductive reactance is very low. This is why an inductor allows the flow of DC and low frequency AC because its reactance decreases with decreasing frequency.

When the frequency is high, the inductive reactance is high. This is why an inductor blocks the flow of high frequency AC because its reactance increases with increasing frequency.

28.4.3 Exercise - capacitance and inductance

1. Describe what is meant by reactance.
2. Define the reactance of a capacitor.
3. Explain how a capacitor blocks the flow of DC and low frequency AC but allows the flow of high frequency AC.
4. Describe what is an inductor
5. Describe what is inductance
6. What is the unit of inductance?
7. Define the reactance of an inductor.
8. Write the equation describing the inductance of a solenoid.
9. Explain that how an inductor blocks high frequency AC, but allows low frequency AC and DC to pass.

28.5 Summary

1. Electrical generators convert mechanical energy into electrical energy.
2. Electric motors convert electrical energy into mechanical energy.
3. There are two types of generators - AC and DC. An AC generator is also called an alternator.
4. There are two types of motors - AC and DC.
5. Alternating current (AC) has many advantages over direct current (DC).
6. Capacitors and inductors are important components in an AC circuit.
7. The reactance of a capacitor or inductor is affected by the frequency of the AC.

28.6 End of chapter exercise

1. [SC 2003/11] Explain the difference between alternating current (AC) and direct current (DC).
2. Explain how an AC generator works. You may use sketches to support your answer.
3. What are the advantages of using an AC motor rather than a DC motor.
4. Explain how a DC motor works.
5. At what frequency is AC generated by Eskom in South Africa?

IEB 2001/11 HG1 - **Work, Energy and Power in Electric Circuits**

Mr. Smith read through the agreement with Eskom (the electricity provider). He found out that alternating current is supplied to his house at a frequency of 50 Hz. He then consulted a book on electric current, and discovered that alternating current moves to and fro in the conductor. So he refused to pay his Eskom bill on the grounds that every electron that entered his house would leave his house again, so therefore Eskom had supplied him with nothing!

Was Mr. Smith correct? Or has he misunderstood something about what he is paying for? Explain your answer briefly.

6. What do we mean by the following terms in electrostatics?
 - A inductance
 - B reactance
 - C solenoid
 - D permeability

Appendix A

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